PHYX412-1 Fall 2008 : Quantum Mechanics I

Homework Assignment 3 : Quantum Measurements

1. Spin One

The angular momentum of a particle carrying one unit of intrinsic angular momentum can be described by a state vector in a space with three orthonormal basis kets $\{|1\rangle, |2\rangle, |3\rangle\}$ in which the matrix elements of the operators measuring angular momentum along the x, y, and z axes (in the L_z basis) are:

$$\hat{L}_x = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad , \quad \hat{L}_y = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix} \quad , \quad \hat{L}_z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad .$$

A. Which pairs of these three measurements can be performed simultaneously without destroying information about each other? Which of them can be measured simultaneously with the operator

$$\hat{L}^2 \equiv \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 \ ?$$

($\hat{L}^2$ is the total squared angular momentum operator).

B. Given an initial state,

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|1\rangle + e^{i\delta}|2\rangle\right)$$

what are the possible outcomes of a measurement of L_z and what probability do they have?

C. Let's say we measure L_z and get $+1$. In the resulting state, compute $\langle L_x \rangle$ and ΔL_x.

D. Find the minimum (for any possible state) value of $(\Delta L_x)(\Delta L_y)$. Which state realizes this minimum value of the product of the uncertainties?

E. A quantum ensemble of a state $|\psi\rangle$ is found to give $L_z = +1$ with probability $P_{+1} = \frac{1}{4}$, $L_z = 0$ with probability $P_0 = \frac{1}{2}$, and $L_z = -1$ with probability $P_{-1} = \frac{1}{4}$. Write down the *most general* normalized state (in the L_z basis) which results in these probabilities.